Lesson 17. – Summary Notes.
The Difference Between Two Population Means.

- Parameter: $\mu_1 - \mu_2$
- Point Estimator: $\bar{X}_1 - \bar{X}_2 = \widehat{\mu_1 - \mu_2}$
- Standard Error of $\bar{X}_1 - \bar{X}_2$ is

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\ .$$

- Two Forms of the Estimated Standard Error

1. No assumption. $S_1^2 = \hat{\sigma}_1^2$, $S_2^2 = \hat{\sigma}_2^2$

$$S_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

2. Assuming $\sigma_1^2 = \sigma_2^2$. $S_p^2 = \hat{\sigma}_1^2$, $S_p^2 = \hat{\sigma}_2^2$

where $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$ and

$$S_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

$\mu_1 - \mu_2$, $\sigma_1^2 = \sigma_2^2$ Assumed TRUE.

Confidence Interval and Hypothesis Test when $\sigma_1^2 = \sigma_2^2$ is assumed TRUE and $S_p^2 = \hat{\sigma}_1^2 = \hat{\sigma}_2^2$.

- $(1-\alpha)100\%$ Confidence Interval to estimate $\mu_1 - \mu_2$ when $\sigma_1^2 = \sigma_2^2$ is assumed TRUE.

$$\bar{X}_1 - \bar{X}_2 \pm t_{\alpha/2 (n_1+n_2-2)} S_{\bar{x}_1 - \bar{x}_2}$$

$$\bar{X}_1 - \bar{X}_2 \pm t_{\alpha/2 (n_1+n_2-2)} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

- To test $H_0: \mu_1 - \mu_2 = D_0$

$$t_{calc} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{S_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$$

When H_0 is true $t \sim t(n_1+n_2-2)$.

$\mu_1 - \mu_2$. No assumptions on σ_1^2 or σ_2^2.

Confidence Interval and Hypothesis Test when no assumptions are made about the equality of σ_1^2 and σ_2^2. A separate variance estimate is used for σ_1^2 and σ_2^2, i.e., $S_1^2 = \hat{\sigma}_1^2$, $S_2^2 = \hat{\sigma}_2^2$.

- $(1-\alpha)100\%$ Confidence Interval to Estimate $\mu_1 - \mu_2$ with no assumptions on σ_1^2 or σ_2^2.

$$\bar{X}_1 - \bar{X}_2 \pm t^*_{\alpha/2\,(df)} \cdot S_{\bar{X}_1 - \bar{X}_2}$$

$$\bar{X}_1 - \bar{X}_2 \pm t^*_{\alpha/2\,(df)} \cdot \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

- To test $H_0: \mu_1 - \mu_2 = D_0$

$$t^* = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{S_{\bar{X}_1 - \bar{X}_2}} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

when H_0 is true $t^* \sim t(df)$

$$df = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$