STATISTICS 2023 | **NAME IN PRINT**______________________

EXAM TWO | **SIGNATURE IN INK**______________________

SPRING 2015 | **CWID IN INK**______________________

TRUE OR FALSE. Answer with a capital T or F. **(4 points each)**

__F__ 1. A discrete random variable has probability on intervals of values, but has no probability on specific values.

__F__ 2. A Binomial random variable is right skewed if the probability of success on one trial is .9.

__T__ 3. For a Poisson random variable the mean and variance are always equal regardless of the value of the parameter for the variable.

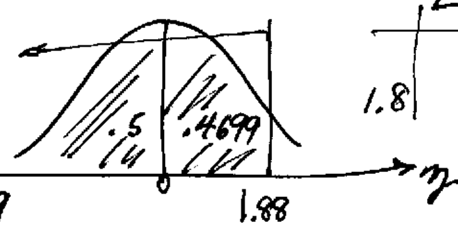
__T__ 4. A variable that has a normal distribution has half of its probability on values that are less than the mean and half of its probability on values that are more than the mean.

__F__ 5. The mean of the sample mean is equal to the mean of the sampled population, but the variance of the set of all possible sample means is the original variance multiplied by n, the number of observations in the sample.

Z-TABLE QUESTIONS. Write the answer on the line. **(4 points each)**

__0.9699__ 6. What is the value of P(Z < 1.88)?

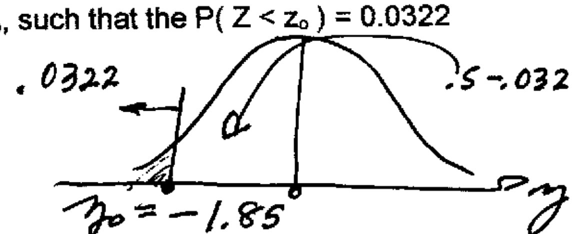
$P(Z < 1.88) =$
$= P(Z < 0) + P(0 < Z < 1.88)$
$= .5 + .4699 = 0.9699$



Z table

	.08
1.8	.4699

__−1.85__ 7. What is the value of z_o, such that the $P(Z < z_o) = 0.0322$

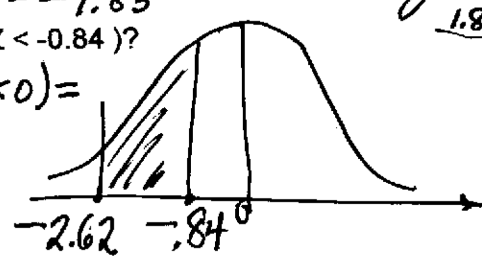


$.5 - .0322 = .4678$

	.05
1.8	.4678

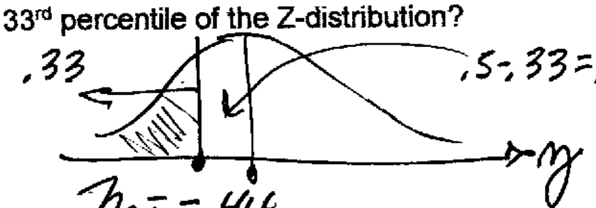
__0.1961__ 8. What is the value of P(-2.62 < Z < -0.84)?

$= P(-2.62 < Z < 0) - P(-.84 < Z < 0) =$
$= .4956 - .2995$
$= 0.1961$



__−0.44__ 9. What is the value of the 33rd percentile of the Z-distribution?

	.04
.4	.1700



$.5 - .33 = .1700$

STATE THE ANSWER. Write the answer on the line. **(4 points each)**

___3___ 10. The possible values of a discrete random variable are x=2, 3, and 6, where the probability on each value is 1/x, so the probability of 2 is ½. What is the expected value of such a variable?

X	2	3	6
$p(x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$EX = \mu = \sum x \cdot p(x) =$

$= 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right) + 6\left(\frac{1}{6}\right) = 3$

___0.40___ 11. Assume a uniform discrete random variable has 10 possible values. What would be the cumulative probability on four of the possible 10 values?

$\sum_{i=1}^{4} p(x_i) = p(x_1) + p(x_2) + p(x_3) + p(x_4) =$

$= .10 + .10 + .10 + .10 = 0.40$

___0.0598___ 12. A new type of car designed by a Japanese car company has a 12% chance of being recalled during the first 3 years of service. Your company plans to purchase a fleet of 8 of these cars, what is the probability that 3 or 4 of them will be recalled during the first 3 years of service? Round your answer to four digits past the decimal.

X = # of cars recalled out of 8

$X \sim Bi(n=8, p=.12)$ also = Bicdf(8,.12,4) − Bicdf(8,.12,2)

$P(X = 3 \text{ or } 4) = P(X=3) + P(X=4) = \binom{8}{3} .12^3 .88^{8-3} + \binom{8}{4} .12^4 .88^{8-4}$

$= 0.0506 + 0.0087046$

___0.5169___ 13. An accounting firm is investigating 20 corporations. The probability that the firm will identify some accounting problem in any one of these corporations is 0.08. What is the probability that the firm will identify accounting problems in fewer than 2 of these corporations? Round your answer to 4 digits past the decimal.

X = # of corporations with accounting problems out of 20

$X \sim Bi(n=20, p=0.08)$

$P(X<2) = P(X \le 1) = Bi\,cdf(20, 0.08, 1) = P(X=0) + P(X=1) =$

$= \binom{20}{0} .08^0 .92^{20} + \binom{20}{1} .08^1 .92^{19} = 0.188693 + 0.32816 = 0.5169$

___0.9561___ 14. The statistics from the NCAA state that the average number of injuries per division one college baseball game is 4.9. What is the probability of at least two injuries in a single game? Round your answer to four digits past the decimal.

X = # of injuries in Division I Baseball game.

$X \sim Poi(\lambda = 4.9)$

$P(X \ge 2) = 1 - P(X \le 1) = 1 - Poi\,cdf(4.9, 1) = 1 - [P(X=0) + P(X=1)] =$

$= 1 - 0.043935 = 0.956065$

___0.9392___ 15. If there are 1.4 vehicle accidents on average in the OSU parking lots in **one** week, what is the probability of at least one accident in a randomly chosen **two**-week time period? Round your answer to four digits past the decimal.

X = # of vehicle accidents in the OSU parking lots in 1 week

$X \sim Poi(\lambda = 2.8)$ for 2-weeks $\lambda = 2.8$

$P(X \ge 1) = 1 - P(X=0) = 1 - Poi\,pdf(2.8, 0) =$

$= 1 - \frac{2.8^0 e^{-2.8}}{0!} = 1 - 0.06081 = 0.93919$

STATE THE ANSWER. Write the answer on the line. **(4 points each)**

On IRS tax forms it is standard practice to round the listed amounts to the nearest dollar. This process ensures that the rounding error averages out to zero. Using cents as the unit, the rounding error would be a uniformly distributed random variable between the values of -50 and +50. Use this information to answer the next three questions. X = Rounding Error. $X \sim Unf(-50, +50)$

0.24 16. What is the probability that the rounding error is more than 38 cents in magnitude?

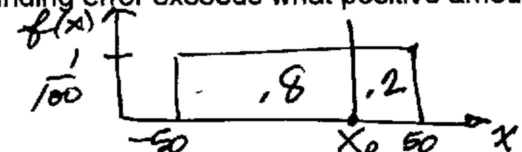
$P(-50<X<-38)+P(38<X<50)=$

$=(-38-(-50))\frac{1}{100} + (50-38)\frac{1}{100} = \frac{24}{100}$

30 17. Twenty percent of the time the rounding error exceeds what positive amount?

$X_0 = 50 - .2(100) = 30$

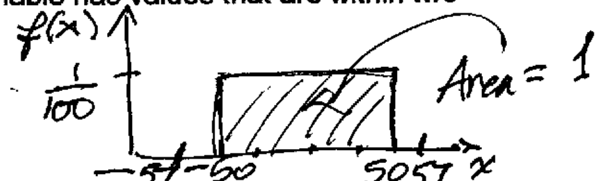
OR $X_0 = -50 + .8(100) = 30$



1 18. What is the probability that this variable has values that are within two standard deviations of the mean? $\mu = 0, \sigma = \sqrt{\frac{(d-c)^2}{12}}$

$P\left(0-2\sqrt{\frac{(50-(-50))^2}{12}} < X < 0+2\sqrt{\frac{(50-(-50))^2}{12}}\right)$

$= P(-57.73 < X < 57.73) = 1$



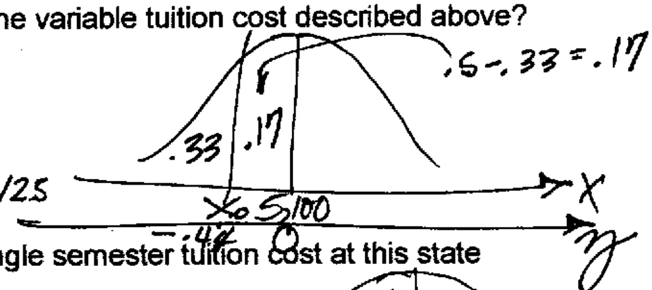
At a large state university the in-state single semester tuition cost for a full-time student is normally distributed with a mean of \$5,100 and a standard deviation of \$125. Use this information to address the remaining questions on this page. X = semester tuition cost, $X \sim N(5{,}100, 125^2)$

\$5,045 19. What is the 33rd percentile for the variable tuition cost described above?

.04; .4 | .1700; $z_0 = -.44$ so

$X_0 = \mu + z_0\sigma = 5{,}100 + (-.44)125 = 5{,}045$

$.5 - .33 = .17$

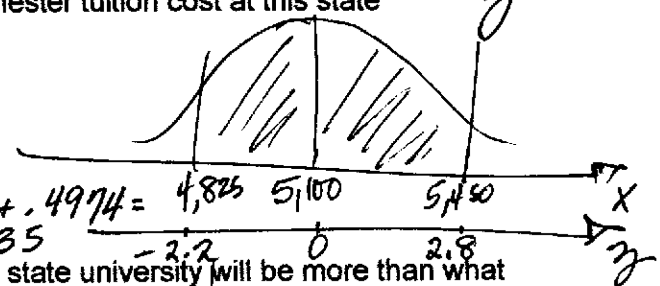


0.9835 20. What is the probability that a single semester tuition cost at this state university will between \$4,825 and \$5,450?

$P(4{,}825 < X < 5{,}450) =$

$= P\left(\frac{4{,}825-5{,}100}{125} < \frac{X-\mu}{\sigma} < \frac{5{,}450-5{,}100}{125}\right)$

$= P(-2.2 < Z < 2.8) = .4861 + .4974 = 0.9835$

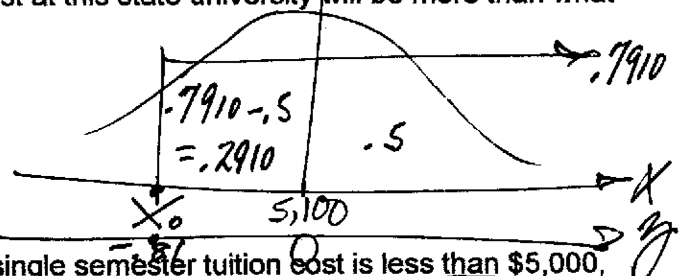


\$4,998.75 21. The single semester tuition cost at this state university will be more than what amount 79.1 percent of the time?

.01; .8 | .2910; $z_0 = -.81$

$X_0 = \mu + z_0\sigma =$

$= 5{,}100 + (-.81)125 =$



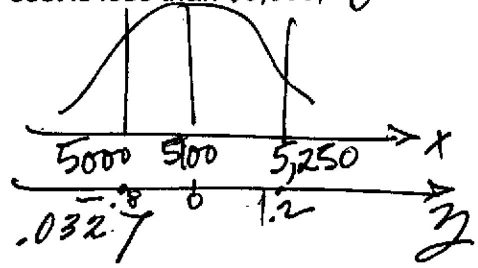
0.0327 22. What is the probability that a single semester tuition cost is less than \$5,000, or more than \$5,250?

$P(X<5000) + P(X>5{,}250)$

$= P\left(\frac{X-\mu}{\sigma} < \frac{5000-5100}{125}\right) + P\left(\frac{X-\mu}{\sigma} > \frac{5250-5100}{125}\right) =$

$= P(Z < -.8) + P(Z > 1.2) =$

$= (.5 - .2881) + (.5 - .3849) = .0327$



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STATE THE ANSWER. Write the answer on the line. (4 points each)

The Chick-fil-a Chicken Sandwich is advertised to have a mean calorie content of 440 calories. Assume the advertising is true and in addition, assume that the standard deviation of the calorie content of the sandwiches is 20 calories. Consider repeated random samples of 100 sandwiches and the resulting set of sample means. Use this sampling description to answer the questions on this page. Repeated Samples of $n = 100$ from $\mu = 440$, $\sigma = 20$

$\bar{X} \sim N\left(\mu_{\bar{X}} = \mu = 440,\ \sigma^2_{\bar{X}} = \frac{\sigma^2}{n}\right)$, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2$

443.92 ______ 23. What is the value of the sample mean of the calorie content of 100 sandwiches that is exceeded by only 2.5% of all the resulting sample means?

What is $\bar{X}_0$, such that $P(\bar{X} > \bar{X}_0) = .025$?

What is z_0, such that $P(Z > z_0) = .025$?

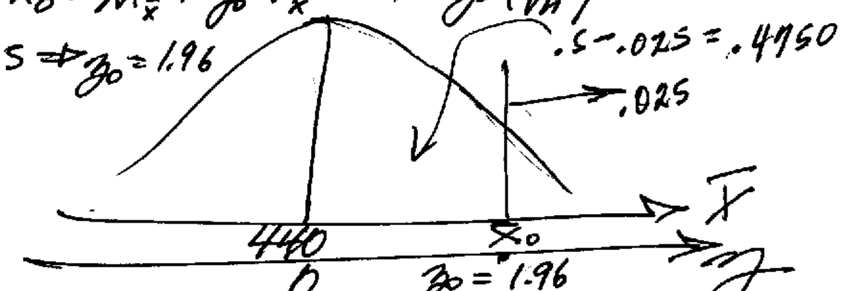
Then Calculate $\bar{X}_0 = \mu_{\bar{x}} + z_0 \sigma_{\bar{x}} = \mu + z_0\left(\frac{\sigma}{\sqrt{n}}\right)$

$P(Z > 1.96) = .025 \Rightarrow z_0 = 1.96$

$\bar{X}_0 = \mu + z_0\left(\frac{\sigma}{\sqrt{n}}\right) =$

$= 440 + (1.96)\frac{20}{\sqrt{100}} =$

$= 443.92$



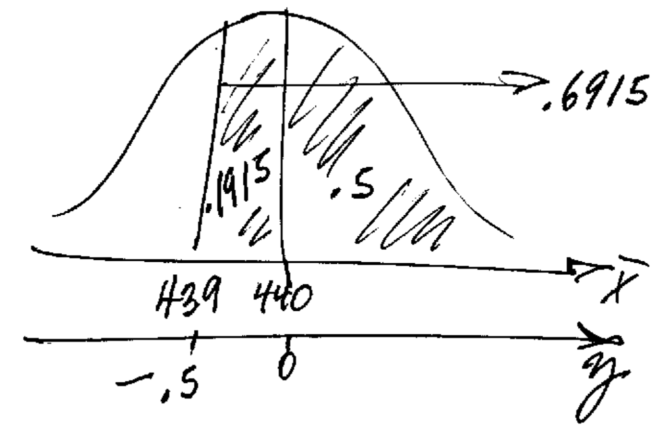
0.6915 ______ 24. What is the probability that the sample mean of the calorie content of 100 Chick-fil-a Chicken Sandwiches exceeds 439?

$P(\bar{X} > 439)$

$= P\left(\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} > \frac{439 - 440}{\frac{20}{\sqrt{100}}}\right)$

$= P(Z > -.5) =$

$= .1915 + .5 = .6915$



437.52 ______ 25. The sample mean of the calorie content of 100 Chick-fil-a Chicken Sandwiches is less than what value 10.75% of the time?

What is the value of $\bar{X}_0$, such that $P(\bar{X} < \bar{X}_0) = .1075$?

1. What is the value of z_0, such that $P(Z < z_0) = .1075$?
2. Calculate $\bar{X}_0 = \mu_{\bar{x}} + z_0 \sigma_{\bar{x}} = \mu + z_0\left(\frac{\sigma}{\sqrt{n}}\right) =$

$= 440 + (-1.24)\left(\frac{20}{\sqrt{100}}\right)$

$= 437.52$