STAT 2023 Exercise Lesson 14

First Exercise on Lesson 14

Z Hypothesis Tests

Name in clear print ______________________

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Suppose an area with horizontal oil wells on each section of land has production that is thought to have a mean of 12 barrels of oil per day per well. Assume the standard deviation of the population is .6.

Case One: A sample of 36 of these wells produced a mean of 11.6 barrels per day. Does this sample provide evidence that the mean daily production differs from 12 barrels?

Case Two: A sample of 36 of these wells produced a mean of 12.05 barrels per day. Does this sample provide evidence that the mean daily production differs from 12 barrels?

Case Three: A sample of 36 of these wells produced a mean of 11.8 barrels per day. Does this sample provide evidence that the mean daily production differs from 12 barrels?

1. What is the set of hypotheses, both null and alternative, for the 3 cases?

$H_0: \mu = 12$ vs. $H_A: \mu \neq 12$

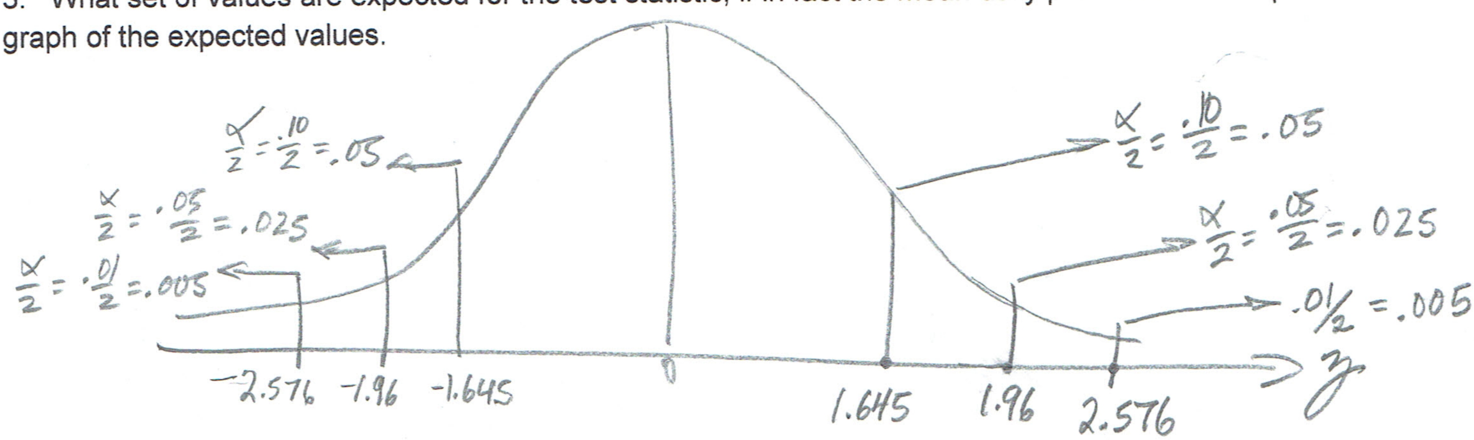
2. What are the observed sample mean and the resulting Z test statistic value for each case?

Case 1: $\bar{X}_1 = 11.6, \; z = \frac{\bar{X}-\mu_0}{\sigma_{\bar{x}}} = \frac{\bar{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{11.6-12}{\frac{.6}{\sqrt{36}}} = \frac{-.4}{.1} = -4$

Case 2: $\bar{X}_2 = 12.05, \; z = \frac{\bar{X}-\mu_0}{\sigma_{\bar{x}}} = \frac{\bar{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{12.05-12}{\frac{.6}{\sqrt{36}}} = \frac{.05}{.1} = .5$

Case 3: $\bar{X}_3 = 11.8, \; z = \frac{\bar{X}-\mu_0}{\sigma_{\bar{x}}} = \frac{\bar{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{11.8-12}{.6/\sqrt{36}} = \frac{-.2}{.1} = -2$

3. What set of values are expected for the test statistic, if in fact the mean daily production is 12bpd? Draw the graph of the expected values.



4. For a two-tail test where are the unlikely values in the Z distribution for α=.10, .05, and .01? Place these values on the graph above.

The values of z below −1.645 and above 1.645 have 10% prob.
The values less than −1.96 and above 1.96 have only 5% probability.
The values less than −2.576 and above 2.576 have only 1% probability.

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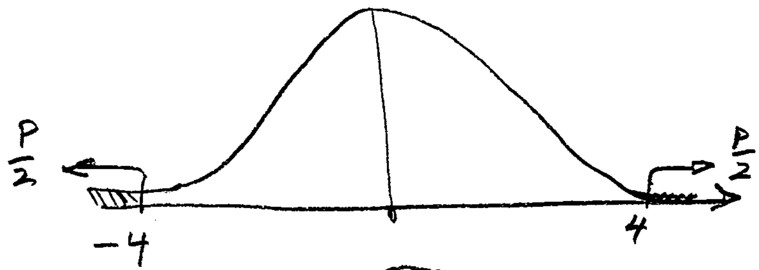
5. For each of the 3 cases calculate the P-value of the hypothesis test. In a two-tail test the P-value is the tail areas associated with the positive and negative values of the test statistic.

Case 1:

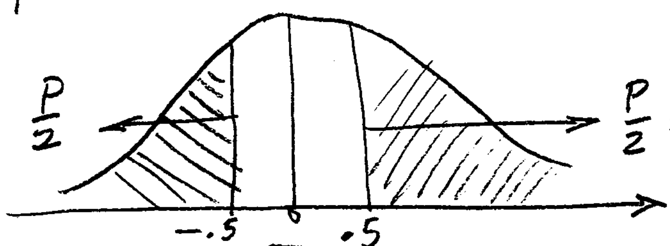


$\frac{P}{2} = 1 - .99997 = .00003$

$P = 2(.00003) =$

$P = .00006.$

Case 2:

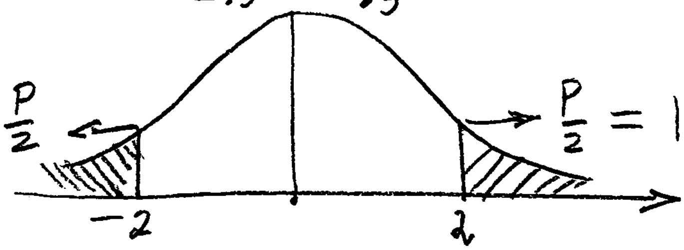


$\frac{P}{2} = 1 - .69146 = .30854$

$P = 2(.30854) =$

$P = .61708.$

Case 3:



$\frac{P}{2} = 1 - .97725 = .02275$

$P = 2(.02275) =$

$= .04550.$

6. For each of the 3 cases identify the set of alpha, α, values for which the null hypothesis that the mean daily production is 12bpd will be rejected.

Case 1: $P = .00006 < .01 < .05 < .10 \Rightarrow$ Reject Ho at .10, .05 and .01

Case 2: $P = .61708 > .10 \Rightarrow$ Do not Reject Ho at any level

Case 3: $P = .04550$

$.01 < .04550 < .05 \Rightarrow$ Reject Ho at $\alpha = .05$
Do not Reject Ho at $\alpha = .01$

7. Construct a 95% confidence interval from each sample mean to estimate the mean daily oil production.

$\bar{x} \pm z_{\alpha/2}\sigma_{\bar{x}} \Rightarrow \bar{x} \pm z_{\frac{.05}{2}} \sigma/\sqrt{n} \Rightarrow \bar{x} \pm 1.96(.1) \Rightarrow \bar{x} \pm .196$

Case 1. $11.6 \pm .196 \Rightarrow (11.404, 11.796) \Rightarrow \mu \neq 12$

Case 2. $12.05 \pm .196 \Rightarrow (11.854, 12.246) \Rightarrow$ Do not Reject $\mu = 12$

Case 3. $11.8 \pm .196 \Rightarrow (11.604, 11.996) \Rightarrow \mu \neq 12$

Any value not contained in the confidence interval would be rejected in a 2-tail test at the same α.