STAT 2023 Exercise Lesson 18.2 **Name in clear print** Key

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See the example beginning on page 9 of 14 of Lesson 18 for the complete illustration of this example.

Suppose we are interested in predicting or estimating starting salaries from college grade-point averages (GPAs). We sampled seven people asking each of them their GPA and their starting salary. The bivariate data, the (x, y) data points, appear as follows:

x(GPA)	y(Salary)	x^2	y^2	xy
2.58	11.5	6.6564	132.25	29.67
3.27	13.8	10.6929	190.44	45.126
3.85	14.5	14.8225	210.85	55.825
3.50	14.2	12.25	201.64	49.7
3.33	13.5	11.0889	182.25	44.955
2.89	11.6	8.3521	134.56	33.524
2.23	10.6	4.9729	112.36	23.638
$\sum x=21.65$	$\sum y=89.7$	$\sum x^2=68.8357$	$\sum y^2=1163.75$	$\sum xy=282.438$

1. What is the corrected sum of squares for the x-variable?

$$SS_x = \sum x^2 - \frac{(\sum x)^2}{n} = 68.8357 - \frac{21.65^2}{7} = 1.875$$

2. What is the corrected sum of squares for the y-variable?

$$SS_y = \sum y^2 - \frac{(\sum y)^2}{n} = 1{,}163.75 - \frac{89.7^2}{7} = 14.3$$

3. What is the corrected sum of squares for the cross product?

$$SS_{xy} = \sum xy - \frac{\sum x(\sum y)}{n} = 282.438 - \frac{21.65(89.7)}{7} = 5$$

4. What is the least squares estimate of the slope in the linear regression equation to estimate the starting salary from GPA?

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_x} = \frac{5}{1.875} = 2.67$$

5. What is the least squares estimate of the y-intercept in the linear regression equation to estimate the starting salary from GPA?

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = \frac{89.7}{7} - 2.67\left(\frac{21.65}{7}\right) = 4.55$$

6. Write the estimated regression equation to estimate the starting salary from the GPA.

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 4.55 + 2.67x.$$

7. Use the estimated regression equation to estimate the average starting salary of students who have a 3.0 GPA.

$$\hat{y}_{x=3.0} = 4.55 + 2.67(3.0) = 12.57$$

8. What is the estimated linear correlation? See page 9 of Lesson 18 for equation.

$$r = \frac{SS_{xy}}{\sqrt{SS_x(SS_y)}} = \frac{5}{\sqrt{1.875(14.3)}} = .97$$ $\Rightarrow$ strong, positive correlation.

9. What is the estimated variance of the residuals? See page 9 of Lesson 18 for equation.

$$S_\varepsilon^2 = \frac{\Sigma(y-\hat{y})^2}{n-2} = \frac{SS_y - \hat{\beta}_1 SS_{xy}}{n-2} = \frac{14.3 - 2.67(5)}{7-2} = 0.19$$

10. What is the estimated standard error of the slope estimate? See top equation page 10 Lesson 18

$$S_{\hat{\beta}_1} = \sqrt{\frac{S_\varepsilon^2}{SS_x}} = \sqrt{\frac{.18}{1.87}} = .315$$

11. What is the 95% confidence interval that results from these data to estimate the slope?

$$\hat{\beta}_1 \pm t_{.\frac{05}{2}(5)} S_{\hat{\beta}_1}$$

$2.67 \pm 2.571(.315)$

$2.67 \pm .809865$

$(1.86, 3.48)$

12. What is the set of hypotheses to test whether GPA affects starting salary?

$H_0: \beta_1 = 0$

$H_A: \beta_1 \neq 0$

13. What is the test statistic to test the above null hypothesis

$$t = \frac{\hat{\beta}_1 - \beta_{1_0}}{S_{\hat{\beta}_1}} = \frac{2.67 - 0}{.31} = 8.613$$

14. What is the set of hypotheses to test whether a unit increase in GPA results in an average increase of more than \$4,000 for starting salary?

$H_0: \beta_1 = 4,000$

$H_A: \beta_1 > 4,000$

15. What is the test statistic to test the above null hypothesis?

$$t = \frac{\hat{\beta}_1 - \beta_{1_0}}{S_{\hat{\beta}_1}} = \frac{2.67 - 4}{.31} = -4.29$$

16. What is the conclusion of the test above?

P-value is $P(t(5) > -4.29)$

The p-value is large (almost 1) so Ho is definitely not rejected.