Hypothesis Test on $\mu_1 - \mu_2$

1. $H_0: \mu_1 - \mu_2 = 0$ OR $H_0: \mu_1 = \mu_2$

$H_A: \mu_1 - \mu_2 > 0$ $\qquad$ $H_A: \mu_W > \mu_A$

2. $$t^* = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{S_{\bar{X}_1 - \bar{X}_2}} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= \frac{285 - 264 - 0}{\sqrt{\frac{400}{40} + \frac{600}{40}}} = \frac{21}{5} = 4.2$$

3. If H_0 is true, $\mu_1 = \mu_2$ then

$$t^*_{calc} \,\dot\sim\, t(df), \text{ where } df = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$$df = \frac{\left(\frac{400}{40} + \frac{600}{40}\right)^2}{\frac{\left(\frac{400}{40}\right)^2}{40-1} + \frac{\left(\frac{600}{40}\right)^2}{40-1}} = 75$$

$t^*(75)$ is estimated well with Z-dist.

Page 195

Water-Cooled (1)	Air-Cooled (2)
$n_1 = 40$	$n_2 = 40$
$\bar{X}_1 = 285$	$\bar{X}_2 = 264$
$S_1^2 = 400$	$S_2^2 = 600$

Do the data indicate that the mean cost of cutting \$1,000 worth of stone is more for the water-cooled method than for the air-cooled method?

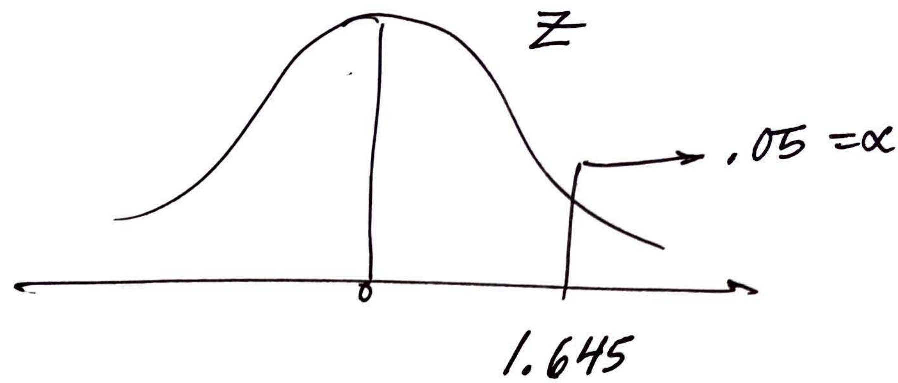
Address the above question and do not assume that $\sigma_1^2 = \sigma_2^2$.

That is,

$$\hat{\sigma}_{\bar{X}_1 - \bar{X}_2} = S_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

4. Decision is to reject or not reject H_0.

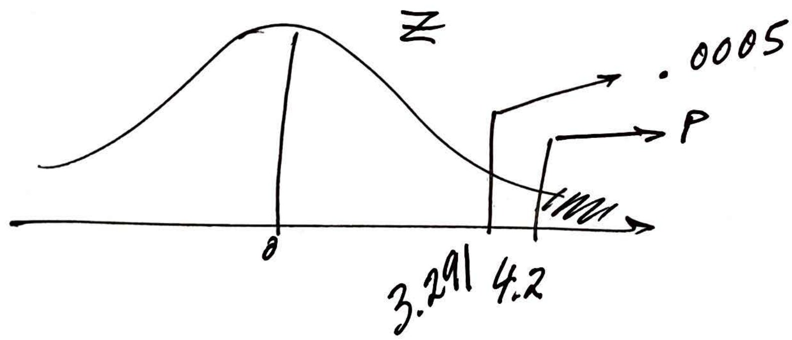
$\alpha = .05$



Reject H_0 at $\alpha = .05$ if $t^* > 1.645$.

Since $t^* = 4.2 > 1.645 \Rightarrow$ Reject H_0.

P-value $= P(Z > 4.2)$



$P < .0005 \Rightarrow$ Strongly Reject H_0.

Conclude, the data indicate that the mean cost of cutting \$1,000 worth of stone is more for the water-cooled method than air-cooled method.

95% confidence interval to estimate $\mu_1 - \mu_2$

(not assuming $\sigma_1^2 = \sigma_2^2$)

$$\bar{X}_1 - \bar{X}_2 \pm z_{\frac{.05}{2}} \cdot S_{\bar{X}_1 - \bar{X}_2}$$

$$\bar{X}_1 - \bar{X}_2 \pm z_{.025} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$285 - 264 \pm 1.96\,(5)$$

$$21 \pm 9.8$$

$$(11.20,\ 30.8)$$